QUEENS COLLEGE
Department of Mathematics
Final Examination
$2\frac{1}{2}$ Hours

Mathematics 152 Fall 2024

<u>Instructions:</u> Answer each question. Show your work and justify your answers. Partial credit will be awarded for relevant work.

1. Compute the following limits. If a limit is $+\infty$, $-\infty$ or does not exist, state this as your answer and explain why.

   (a) $\lim_{x\to 0^+} \dfrac{\sin(x^2+x)}{\sqrt{x}}$

   (b) $\lim_{x\to\infty} x(\tan^{-1} x - \frac{\pi}{2})$

   (c) $\lim_{x\to 0^-} \dfrac{e^x}{\ln(x^2+1)}$

2. Let $f(x) = \ln(e^{-2x+1}+1)$.

   (a) Show that $f(x)$ is a one-to-one function using an appropriate calculation. (No calculator allowed for this problem!)

   (b) Let $f^{-1}(x)$ be the inverse of $f(x)$. Find $(f^{-1})'(\ln 2)$ using the formula for $(f^{-1})'(x)$ in terms of the derivative of $f(x)$.

3. Let R be the region in the first quadrant bounded by the graph of $y = \cos x$ and the coordinate axes.

   (a) Find the volume of the solid of revolution obtained when R is rotated about the y-axis.

   (b) Find the volume of the solid of revolution obtained when R is rotated about the line $y = -1$.

4. Evaluate the following integrals:

   (a) $\displaystyle\int \frac{3x^2-3x+1}{x^2+1}dx$

   (b) $\displaystyle\int \frac{-9x+3}{(x-2)(x^2-1)}dx$

5. Evaluate the following improper integrals, if they converge. If an integral is divergent, explain why.

   (a) $\displaystyle\int_0^\infty \frac{1}{4+x^2}dx$

   (b) $\displaystyle\int_0^1 \ln x dx$

6. Evaluate the following integral:

$$\int \frac{dx}{\sqrt{4x^2-9}}$$

(continued on the back)

7. Determine whether each of the following sequences is convergent or divergent. If a sequence converges, find its limit; if a sequence diverges, explain why.

   (a) $a_n = \dfrac{e^{2n+3}}{e^{n^2}}$

   (b) $a_n = \sin \dfrac{n\pi}{2}$

8. Using an appropriate test, determine the convergence or divergence of each of the following series. In the case of convergence, classify the convergence as either absolute or conditional.

   (a) $\sum_{n=0}^{\infty} \dfrac{(-1)^{n+1}}{2n+5}$

   (b) $\sum_{n=1}^{\infty} \tan^{-1}(\dfrac{1}{n})$

   (c) $\sum_{n=2}^{\infty} \dfrac{1}{n(\ln n)^{3/2}}$

9. Find the interval of convergence for the power series

$$\sum_{n=0}^{\infty} (-1)^n \frac{n3^n}{n^3+1} x^n.$$

10. Let $f(x) = \frac{1}{x^2}$.

   (a) Find the Taylor series for f at $a = 2$ and determine its radius of convergence.

   (b) Use $R_4(x) = |f(x) - T_4(x)|$ to estimate the largest possible error that can result when $T_4(x)$, the fourth Taylor polynomial of f at $a = 2$, is used to estimate $f(x)$, when $1 \le x \le 3$.