Queens College
Department of Mathematics
Final Examination
$2\frac{1}{2}$ Hours

Instructions:

- ***Read each problem carefully.*** Make sure you understand what the problem is asking.
- **You must show all your work.** No credit will be given to a problem without work. For incorrect solutions, partial credit will be given where appropriate.
- All your work and solutions must be recorded in the provided Blue Book. Additional Blue Books are available if necessary.
- You are allowed to use a calculator that is in accordance with the calculator policy set by the mathematics department. You may not share calculators during the exam.
- No notes or devices other than a writing utensil and calculuator may be used during the exam.
- Unless otherwise noted, answers must be precise, not approximate (for example, $\frac{1}{3} \neq 0.33$).

1. Compute the following limits or state the limit does not exist.

   (a) $\displaystyle\lim_{x\to 3} \frac{x^3 - 9x}{x^2 - 5x + 6}$ (b) $\displaystyle\lim_{x\to 2} \frac{x^2 + 4x - 5}{x - 2}$ (c) $\displaystyle\lim_{x\to\infty} \frac{\sqrt{8x^6 - 3x^3 - 1}}{2x^3 - 10x^2 + 5}$

2. Use the limit definition of the derivative to compute the derivative of $f(x) = \sqrt{3x}$.

3. Suppose f is a function given by $f(x) = 3x^2 - 2x + c$ for some number c and suppose that the line $y = 4x + 5$ is the tangent line to $f(x)$ at $x = a$. Find the numbers a and c.

4. In each of the following, compute $\frac{dy}{dx}$:

   (a) $y = \cos(3x^4 - 10x^2 + 5)$

   (b) $y = \dfrac{x+1}{x^2 - 1}$

   (c) $x^2y^2 = x^3 - xy$

   (d) $y = \displaystyle\int_0^{\sin x} \sqrt{t^2 + 1}\, dt$

5. You have 1530 feet of fencing. You need to build two enclosures, a square enclosure and a rectangular enclosure where the length is twice the width. What is the maximum area that can be enclosed in this configuration?

6. The tension T (in Newtons) of a guitar string is related to the frequency f (in Hertz) of the note heard when plucked by the formula $T = kf^2$ for some constant k.

   (a) When $f = 320$ Hz, T is measured to be 60 N. Find the constant k. (This is pretty close to accurate for the high E string on a guitar.)

   (b) If the tension of the string is decreased at a constant rate of 2 Newtons per second, find the rate the frequency is decreasing (in Hertz per second) when $T = 310$ N.

(continued on the back)

7. Suppose $f(x)$ is a differentiable function whose domain is all real numbers and that has the following properties:

- $\lim_{x \to -\infty} f(x) = -\infty$
- $\lim_{x \to \infty} f(x) = 0$
- f has a two-sided vertical asymptote at $x = 0$
- $f'(x) > 0$ on $(-\infty, -2)$ and on $(1, \infty)$
- $f'(x) < 0$ on $(-2, 0)$ and on $(0, 1)$

(a) Find the critical numbers of $f(x)$, if any.

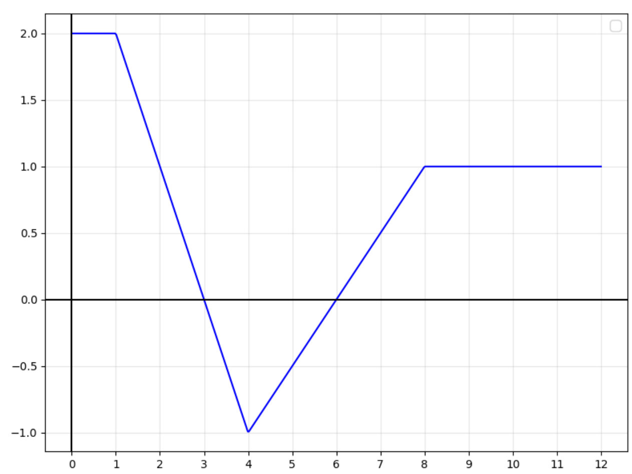
(b) Find the intervals on which $f(x)$ is decreasing, if any.

(c) Find the x values of the local maxima and the local minima of $f(x)$, if any.

(d) Sketch the graph of $f(x)$.

(e) Now assume that $f''(x)$ exists for every value of x and that the only solution to $f''(x) = 0$ is $x = 2$. Find the intervals on which f is concave up and the intervals on which it is concave down. (The graph you drew should be helpful!)

8. Below is the graph of the function $f(x)$. In the question below, $A(x)$ is the function defined by $A(x) = \int_0^x f(t)\,dt$.



(a) Evaluate $A(3)$.

(b) Evaluate $A(8)$.

(c) Find the local maxima and minima of A, if any.

(d) On the interval $[1, 4]$, is A concave up, concave down, or neither? Explain.

9. Find each of the following integrals:

(a) $\int_1^2 (8x^3 - 6x^2 + 4x)\,dx$

(b) $\int 3\theta \cos(\theta^2 + 2)\,d\theta$

(c) $\int x\sqrt{x-5}\,dx$

10. A particle moves with acceleration $a(t) = \sqrt{t}$ meters per second squared and with an initial velocity of 10 meters per second. How far has the particle moved after 100 seconds?