

**Queens College
Department of Mathematics**

Final Examination

$2\frac{1}{2}$ Hours

Mathematics 143

FALL 2024

Instructions: Answer all questions and show all your work.

1. Evaluate the limit: $\lim_{x \rightarrow 0^+} [\cos(3x)]^{\frac{1}{x}}$

2. Evaluate the integrals.
 - (a) $\int e^x \sin(2x) dx$
 - (b) $\int x^3 \sqrt{x^2 + 16} dx$
 - (c) $\int \frac{2x^2 - x + 20}{(x - 2)(x^2 + 9)} dx$
 - (d) $\int_2^3 \frac{x}{x^2 - 4} dx$
 - (e) $\int_0^\infty \frac{dx}{9x^2 + 4}$

3. Determine whether the series is conditionally convergent, absolutely convergent or divergent.
 - (a) $\sum_{n=0}^{\infty} \frac{2^{n+1}}{3^{n-1}}$
 - (b) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n}}$
 - (c) $\sum_{n=1}^{\infty} \frac{2n^3 - 1}{5n^3 + 1}$
 - (d) $\sum_{n=1}^{\infty} \frac{n^3 - 1}{4n^5 + 2}$

4. Find the radius and interval of convergence for the power series $\sum_{n=1}^{\infty} \frac{(-1)^n (x - 3)^n}{3^n \sqrt{n}}$.

5.
 - (a) Using the power series for e^x , find a power series for xe^{-x^3} .
 - (b) Using part (a) find $\int_0^{0.5} xe^{-x^3} dx$ so the error is correct to 4 decimal accuracy.

6. Let $f(x) = \ln(x + 2)$.
 - (a) Find $T_4(x)$, the fourth Taylor polynomial, centered at $a = 1$.
 - (b) Use part (a) to approximate the value of $\ln(3.2)$. ($x = 1.2$)
 - (c) Find an estimate for the error when part (b) is used to approximate using Taylor's Theorem.

7.
 - (a) Find a power series representation for $f(x) = \frac{1}{1 + x^3}$.
 - (b) Tell whether the alternating series $\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{n+2}{n^5}\right)$ is convergent or divergent. If it is convergent, find an estimate for the sum of the series correct to two decimals.