

**Queens College**  
**Department of Mathematics**

**Final Examination**

**$2\frac{1}{2}$  Hours**

**Mathematics 142**

**FALL 2024**

**Instructions: Answer all questions and show all your work.**

1. Find  $\frac{dy}{dx}$ : (algebraic simplification not necessary)

(a)  $y = e^{x^3 + \tan x}$

(b)  $y = \int_{x^2}^{x^3} \frac{t^2 - 1}{t^2 + 2} dt$

(c)  $y = \frac{(x^2 + x)^3 \cdot \sin^{-1} x}{\sin^4 x \cdot e^{\cos x}}$

(d)  $y = \ln(\tan(x))$

(e)  $y = \frac{1}{\ln x}$

2. Evaluate:

(a)  $\int \frac{\sec x \tan x}{e^{\sec x}} dx$

(b)  $\int \pi^x dx$

(c)  $\int_1^e \frac{1}{[1 + (\ln x)^2] \cdot x} dx$  (Find exact value, calculator solution not acceptable)

(d)  $\int \frac{e^{2x}}{\sqrt{1 - e^{4x}}} dx$

3. Penicillin-V is eliminated from the body at a rate proportional to the amount present.

(a) How much will remain in the body  $2\frac{1}{2}$  hours after a patient takes a 500 mg dose? (The half-life of Penicillin-V is 3.7 hours)

(b) When will the patient have 0.01 mg Penicillin left in the body?

4. (a) Find the function  $g$  such that  $g(f(x)) = x$  if  $f(x) = \frac{x+1}{x-1}$ .

(b) Find the value of  $\sin\left(\tan^{-1}\frac{a}{b}\right)$ . (Answer will be in terms of  $a$  and  $b$ .)

5. The slope of a curve is given by  $y^2(1-x)$ . The curve passes through the point  $(0,1)$ . Find an equation of the curve.

6. Find the exact area between the curves  $y = x - 3$  and  $x = y^2 + 1$  (calculator solution not acceptable).

7. For each of the following, set up the integral(s) which will give the volume generated when the region is rotated. Do not evaluate the integrals.

- (a) Region I is revolved around the  $x$ -axis.  
 (b) Region II is revolved around the  $y$ -axis.  
 (c) Region III is revolved around the line  $y = 20$ .

