QUEENS COLLEGE
Department of Mathematics
Final Examination
$2\frac{1}{2}$ Hours

Mathematics 141 Fall 2024

Directions: Read each question on this exam before you start working so you can get the flavor of the questions. Please show all of your work. Unsupported answers will not even be graded. Anyone who is caught cheating will face academic and disciplinary sanctions according to CUNY policy on academic integrity.

1. Without using your calculator, find the following limits. (*Note: If a limit is $+\infty$, $-\infty$, or does not exist, state this as your answer.*)
   (a) $\lim_{x\to -3} \dfrac{x^2-9}{x^2+2x-3}$
   (b) $\lim_{h\to 0} \dfrac{(h-1)^2+1}{h}$.
   (c) $\lim_{x\to \infty} \dfrac{1+2x-x^2}{1-x+2x^2}$.
   (d) $\lim_{x\to 0^-} \dfrac{\sin(3x)}{\sin(4x)}$.

2. Using the definition of derivative (this involves a limit!), find $f'(x)$ where $f(x) = \frac{3}{x^2}$.

3. In each of the following, solve for $\frac{dy}{dx}$ in terms of x and y (you need not simplify).
   (a) $y = (5x^2 - 2x + 9)^4 \tan(x^2)$.
   (b) $y = \frac{\sin(x)}{1+\sin(x)}$.
   (c) $y = (x\ \cos(x))^{1/5}$.
   (d) $x\ \sin(y) = y + 1$.

4. (a) Show that the equation $2\sin(x) = 3 - 2x$ has a root in the interval $(0, 1)$, using the Intermediate Value Theorem.
   (b) Use your graphing calculator to find the root in part (a), correct to three decimal places.

5. A car has position function $s(t) = t^3\cos(t)$.
   (a) Determine the average velocity of the car on the interval $[\pi/4, \pi/2]$.
   (b) Determine the instantaneous velocity of the car at time $t = \pi/2$.

6. A paper cup has the shape of a cone with height 10 cm and radius 3 cm (at the top). If water is poured into the cup at a rate of 2 cm^3/s, how fast is the water level rising when the water is 6 cm deep? Recall that the volume of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.

7. Let $f(x) = x^3 - 6x^2 - 15x + 4$. Use calculus and show all necessary steps, to
   (a) find the intervals of increase and intervals of decrease of f
   (b) find the local maximum and minimum values of f, if any
   (c) find the intervals where f is concave up and those where f is concave down
   (d) find any and all inflection points of f.
   (e) Use the information found in parts a) through d) to sketch the graph of $y = f(x)$.

8. If 7500 cm^2 of material is available to make a box with a square base and an open top, find the largest possible volume of the box.