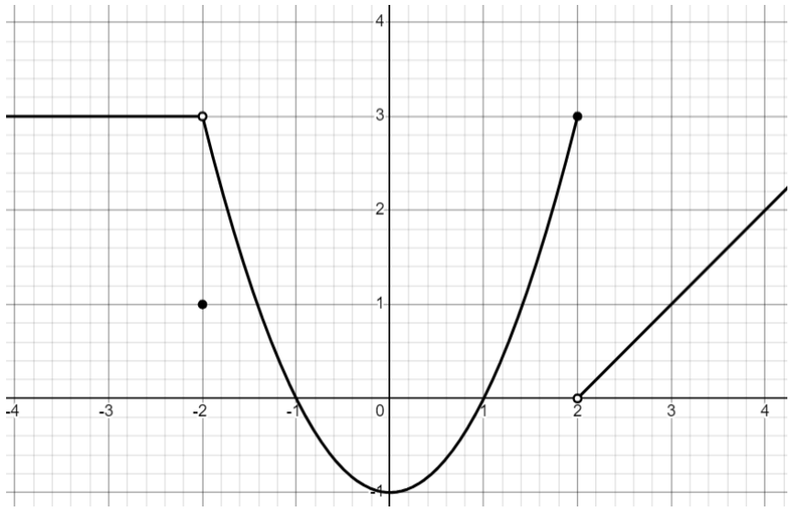
QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS

Final Examination
2 ½ Hours

Mathematics 131 **Fall 2024**

Instructions: **Answer all questions.** **Show all work.**

1. Use the provided graph of $f(x)$ to evaluate each of the following:



a) $\lim_{x \to -2^+} f(x)$

b) $\lim_{x \to -2^-} f(x)$

c) $\lim_{x \to -2} f(x)$

d) $f(-2)$

e) $\lim_{x \to -\infty} f(x)$

f) $f(2)$

g) $\lim_{x \to 2^+} f(x)$

h) For what value(s) of x does the function f have a discontinuity? Explain using the definition of continuity.

2. Evaluate each limit. ($+\infty$, $-\infty$, and DNE are acceptable possible answers.)

a) $\lim_{x \to 2} \dfrac{x^2 - 4}{x^2 + 3x - 10}$

b) $\lim_{x \to 9} \dfrac{\sqrt{x} - 3}{x - 9}$

c) $\lim_{x \to +\infty} \dfrac{-2x^5 + x^3}{5x^2 - 1}$

d) $\lim_{x \to 3} f(x)$, given $f(x) = \begin{cases} x + 2 & \text{if } x \leq 0 \\ x^2 - 1 & \text{if } x > 0 \end{cases}$

3. Given $f(x) = 5x^2 + 2x - 10$.
   a) Use the definition of the derivative to find $f'(x)$.
   b) Find an equation of the line tangent to the graph of $f(x)$ at the point where $x = 0$.

4. Given $f(x) = \frac{1}{3}x^3 - 4x + 7$.
   a) Can the Intermediate Value Theorem be used to show $f(x) = 5$ for some value of x in the interval (0, 3)? Justify your answer.
   b) Find the absolute extrema of $f(x)$ on the interval $0 \leq x \leq 3$.

(continued on the back)

5. Find $\frac{dy}{dx}$ for each of the following functions. You do not need to simplify.

a) $y = \frac{4x^2}{3} + \sqrt[5]{x^3} - 7e^x + \ln(2x^4 + x) + \pi$

b) $y = \frac{\sqrt{3x+1}}{e^{4x}}$

c) $y = 3^{x^2+5x}$ (Hint: Use logarithmic differentiation.)

d) $3xy^2 - y^5 + 10 = x^3$

6. a) Suppose $2500 is invested into a bank that pays an annual interest rate of 3.5%. Compute the balance after 5 years if interest is compounded
   i) semi-annually.
   ii) continuously.

   b) How much money should be invested at an annual interest rate of 3% compounded continuously so that it will be worth $2000 in six years?

7. A manufacturer will produce and sell x units of a product when the price is $p = x^3 + 2x$ dollars. The total cost to produce x units is given by the function $C(x) = 3x^3 + 2x + 500$ dollars.
   a) Use marginal analysis to estimate the cost of producing the11th unit.
   b) Find the actual cost of producing the 11th unit.
   c) Find the average cost per unit if 10 units are produced.
   d) Find the total revenue function, $R(x)$.
   e) Find the total profit function, $P(x)$.

8. Let $f(x) = \frac{1}{3}x^3 + 2x^2 + 3x + 2$. Using calculus (not your graphing calculator):
   a) Find the intervals of increase and decrease of f.
   b) Find the relative (local) maxima and minima of f and their coordinates.
   c) Find the intervals of upward and downward concavity of f
   d) Find the inflection point(s) of f and their coordinates.
   e) Using the information from parts a - d, carefully sketch the graph of f. Label all relative extrema, inflection points, and the y-intercept.

   Be sure to clearly label each part of the problem you are working on

9. When Monkey D. Luffy uses his Gear 4 attack, he expands his body into an approximately spherical shape ($V = \frac{4}{3}\pi r^3$). If the radius of the sphere is increasing at a rate of 0.5cm/s, find the rate at which the volume of the sphere is increasing at the instant when the radius is 20cm.