Mathematics 122   **Fall 2024**
Instructions: Answer all questions. Show all work.

1. Consider the piecewise function
$$f(x) = \begin{cases} -3 & \text{if } x < -5 \\ x^2 & \text{if } -3 \le x < 2 \\ x - 1 & \text{if } 4 \le x \end{cases}$$

    Evaluate $f(5)$, $f(-6)$, and $f(-2)$.

2. Find the domain of $f(x) = \dfrac{x+1}{2x^2 - 11x + 5}$. Express your answer in interval notation.

3. Given $f(x) = 3x^2 + 4x - 5$, find and simplify:
    a) $f(a)$
    b) $f(a+h)$
    c) The difference quotient, $\dfrac{f(a+h) - f(a)}{h}$, where $h \ne 0$.

4. a) Given the graph of $y = f(x)$, explain what transformations you would do to obtain the graph of
        (i) $y = f(x+1) + 4$.
        (ii) $y = -f(x) - 1$.

    b) Let $f(x) = \sqrt{x - 3}$.
        (i)   Find the domain of $f(x)$. Express your answer in interval notation.
        (ii) Using appropriate transformations, sketch the graph of $f(x)$. Label the coordinates of all intercepts of the graph.

    c) Starting with the graph of $y = g(x) = \dfrac{1}{x}$, use appropriate transformations to sketch the graph of $y = h(x) = \dfrac{1}{x-1} + 2$. Label the coordinates of any and all intercepts of the graph of h and write an equation of any horizontal and vertical asymptotes of the graph of h.

    d) Let $f(x) = |x + 4| - 5$
        (i)   Find the domain of $f(x)$. Express your answer in interval notation.
        (ii) Using appropriate transformations, sketch a graph of $f(x)$, labeling the axes and intercepts.

5. Let $f(x) = 3x^2 - 5$ and $g(x) = 4x - 7$. Find $f \circ g$ and simplify.

6. Consider the function $f(x) = \dfrac{5x + 3}{7x - 2}$.
   a) Find the domain of $f(x)$. Express your answer in interval notation.
    b) Find $f^{-1}(x)$, the inverse function of $f(x)$, and its domain. Express your answer in interval notation.
    c) Evaluate $f^{-1}(1)$.

7. A quadratic function f is given to be $f(x) = x^2 - 6x + 5$. (Only algebraic solutions will be accepted.)
   a) Express f in standard form.
   b) Find the coordinates of the vertex.
   c) Find the x- and y-intercepts of f.
   d) Sketch a graph of f.
   e) Find the minimum or maximum value of f. Is this a minimum or maximum value?

8. Given 120 ft of fencing enclose a rectangular field that is divided into 3 sections by placing fencing parallel to one side of the rectangular field. Find its dimensions so that the entire area of the rectangular field is maximized. What is the maximum area of the rectangular field? Hint: Refer to the drawing below:
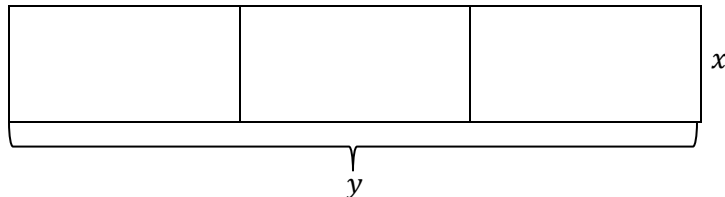
9. Solve each of the following equations for x:
   a) $\left(\frac{1}{8}\right)^{x+2} = 32^{2-x}$

   b) $\log_2(x + 2) + \log_2(x + 3) = 1$

   c) $5^{1-x} = 7$ Round answer to five decimal places.

10. Using the laws of logarithms, NOT A CALCULATOR, evaluate the expression $\log_3 30 - \log_3 50 + \log_3 5$.

11. WITHOUT USING YOUR CALCULATOR,
   a) find the exact value of $\sin\frac{11\pi}{12}$ by using an appropriate addition or subtraction formula.
   b) find the exact value of $\cos\frac{\pi}{9}\cos\frac{\pi}{18} - \sin\frac{\pi}{9}\sin\frac{\pi}{18}$ by using an appropriate addition or subtraction formula.

12. If $\tan\theta = -\frac{3}{5}$ and $\sin\theta < 0$, find
   a) $\sin\theta$
   b) $\cos\theta$
   c) $\sin 2\theta$
   d) $\cos 2\theta$

13. Verify the identity $\sin\theta\tan\theta + \cos\theta = \sec\theta$.

14. Find the amplitude and period of the function $f(x) = 3\sin(2x)$ and graph it in the interval $[0,2\pi]$.

15. Solve for θ, where θ is in the interval $[0,2\pi)$:
$$\cos\theta\,(2\cos\theta + 5) = 3$$