

MATH 120 — Final Exam — 15 December 2024

Name: _____

Class Section: _____

Page 1	
Page 2	
Page 3	
Page 4	
Page 5	
Total Score	
Grade	

Important Information:

WAIT until you are told to start.

For the counting questions, leave your answers in terms of $n!$, $P(n, k)$, $\binom{n}{k}$, and $\binom{n}{k}$.

You are expected to **SHOW YOUR WORK** in all answers.

Wrong answers provided without work will receive no credit.

Every part of every question is worth the same number of points.

You may use the back of pages if you need more space or for scrap work.

If you continue work in a different location and want it to be graded,
indicate **CLEARLY** where it is.

1. **FIRST** write the following statements using symbols and **THEN determine** if the statement is True or False. No explanation is necessary.
 - (a) The empty set is a subset of the set S .

 - (b) The set of all real numbers is a proper subset of the set of all integers.

2. Give an example of two sets that are disjoint and explain why they are disjoint.

3. Let F be the set of all odd multiples of 5.
 - (a) Write F in roster notation.

 - (b) Write F in set-builder notation.

4. (a) **Draw** and **Label** a Venn Diagram representing the set of students at Queens College who are taking Biology, Chemistry, and/or Physics courses.
 - (b) Shade the region(s) of the diagram that correspond to the students who are taking exactly two of those subjects.

5. You are designing a flag with three horizontal stripes. You have 5 different colors to choose from, and you can use a color more than once.
- (a) How many different flags can you design?

 - (b) How many different flags can you design if each color can be used at most once?

 - (c) How many different flags can you design if each color can be used at most twice?
6. Define C to be the set of all bit strings of length 6, D to be the set of bit strings of length 6 with exactly three 1's, and E to be the set of bit strings of length 6 that begin with a 1.
- (a) Determine the number of elements in the set C .

 - (b) Calculate the number of elements in the set $D \cap E$.

 - (c) Write a description in words of the set $C \setminus D$.
7. A pizza place offers 3 different crusts (thin, pan, cheese-filled), 4 different sauces (red, pesto, white, spicy), and 5 different toppings (mushroom, olives, pepperoni, green peppers, anchovies).
- (a) How many different one-topping pizzas can be created?

 - (b) How many different pizzas with thin crust and red sauce can be created with **three** toppings, assuming that toppings **may** be repeated? (In other words: triple pepperoni is allowed.)

--

8. Let Q be the set of students in your MATH 120 class and let M be the set of months in the year. Consider the function $b : Q \rightarrow M$ that maps a student to the month when they were born.
- (a) Is b a well-defined function? Justify your answer.

 - (b) Describe what about the class would need to be true for b to be an **injective** function.

 - (c) Describe what about the class would need to be true for b to be a **surjective** function.
9. Let h be the function that maps a natural number to the sum of its digits. For example, $h(123) = 6$.
- (a) Determine the domain of h .

 - (b) Determine a possible codomain for h .

 - (c) Describe in words the pre-image $P = h^{-1}(12)$.

 - (d) Give two elements from P .
10. What is $\left\lfloor 5.5 + \frac{\lceil 100.01 \rceil}{3} \right\rfloor$? Show your work.

11. Write the exponential equation $100^{-1} = 0.01$ in logarithmic form.
12. Assume a , b , and c are all non-zero.
Simplify $\frac{16a^4 \cdot \sqrt[3]{b} \cdot c^{-10}}{(a \cdot a)(b + b)(c/c)}$ into an expression where a , b and c each occur at most once.
13. Convert $\log_2(10)$ to an expression involving only **base 10** logarithms.
14. Expand $\prod_{k=10}^{13} \log_b(k)$ as a product of terms. Your final answer should not involve \prod notation.
15. Write in sigma notation the infinite geometric series G that starts: $125 + 25 + 5 + \dots$.
16. A theater has 20 rows of seats. The first row has 10 seats, and each subsequent row has 2 more seats than the row before it.
- (a) How many seats are in the 10th row?
- (b) How many seats are in the theater in total?
Use summation techniques that you learned in this class to solve this problem.
17. Rewrite the following sum in product notation. $\sum_{i=34}^{55} \ln\left(\frac{2^i}{i^2}\right)$.

18. Use the technique of repeated squaring to compute $2^{100} \bmod 13$.

19. Justify why the greatest common divisor of two prime numbers is always 1.

20. Use the Euclidean Algorithm to find $\gcd(560, 392)$.

21. (a) Find the prime factorization of 1116.

(b) How many divisors does 1116 have?

Page Score