**QUEENS COLLEGE**
**DEPARTMENT OF MATHEMATICS**
**FINAL EXAMINATION**
**$2\frac{1}{2}$ Hours**

**Mathematics 152** **Spring 2024**

**Instructions: Answer all questions. Show all work.**

**1.** Let $f(x) = -7x^5 - 3x^3 - 1$

**a)** Show that $f(x)$ is a one-to-one function.

**b)** Let $f^{-1}(x)$ be the inverse of $f(x)$. Find $(f^{-1})'(-11)$.

**2.** Let $R$ be the region in the $xy$-plane bounded by the curve $y = \sin(x)$ and the $x$-axis on the interval $0 \leq x \leq \frac{\pi}{2}$.

**a)** Find the exact volume of the solid obtained by rotating $R$ about the $y$-axis.

**b)** Find the exact volume of the solid obtained by rotating $R$ about the line $y = -2$.

**3.** For each of the following functions, find the derivative $y' = \frac{dy}{dx}$:

**a)** $y = \sin^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$

**b)** $y = (\tan(x))^x$

**c)** $y = \ln(\cos^2(e^x))$

**4.** Evaluate the following integrals. Calculator approximations will not be accepted. If an integral diverges, indicate that it is divergent:

**a)** $\int e^{x+\ln(x)}\, dx$

**b)** $\int \frac{1}{x^3 + x}\, dx$

**c)** $\int \frac{\sqrt{4-x^2}}{x^2}\, dx$

**d)** $\int_{-2}^{3} \frac{1}{x^5}\, dx$

**e)** $\int \sqrt{\tan(x)}\, \sec^4(x)\, dx$

**5.** Find the exact value of the following limits:

**a)** $\lim_{x \to 0} \frac{9^x - 6^x}{3x}$

**b)** $\lim_{x \to 0} (\sin(2x))^x$

**(continued on the back)**

**6.** Determine whether the sequence defined by $a_n = \frac{\ln(n^2)}{n+1}$ converges or diverges. If it converges, find the limit.

**7.** Using appropriate tests, determine whether the series diverges, converges, or converges absolutely:

**a)** $\sum_{n=1}^{\infty} \frac{(-1)^{2n}}{n^2\ 4^n}$

**b)** $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n^5}\right)$

**c)** $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+7}$

**8.** Find the interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{9^{n+1}}{3n} x^n$$

**9.** Starting with the Maclaurin series $\ln(1+x) = \sum_{n=0}^{\infty} (-1)^{n-1} \frac{x^n}{n}$ for $|x| < 1$, find the series representation for

$$\int \frac{\ln(1+x^3)}{x^4}\, dx$$

**10.** Let $f(x) = \cos(3x)$

**a)** Find $T_5(x)$, the fifth Taylor polynomial of $f$ centered at $a = \frac{\pi}{6}$.

**b)** Use Taylor's formula to estimate the largest possible error that can result by approximating $f(x)$ by $T_5(x)$ when $\frac{5\pi}{36} \leq x \leq \frac{7\pi}{36}$.