QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS
Final Examination
$2\frac{1}{2}$ Hours

Mathematics 151 **Spring 2024**

Instructions: Answer all questions and show all work

1. Compute the following limits:

   (a) $\lim_{x \to 4} \dfrac{\sqrt{2x+1}-3}{x^2-16}$

   (b) $\lim_{t \to 0} \dfrac{\sin^2(5t)}{\tan(3t^2)}$

   (c) $\lim_{x \to -\infty} \dfrac{\sqrt{4x^6-1}}{5x^3+x}$

   (d) $\lim_{x \to -3} \dfrac{7x+21}{|x+3|}$

2. Use the definition of derivative to find the derivative of the function $f(x) = \sqrt{2x+1}$ at the point $x = 4$.

3. Compute the derivative $\dfrac{dy}{dx}$ for each of the following functions. Do not simplify.

   (a) $y = \sin^2(\cos^2(x))$

   (b) $y = \dfrac{(2x^2+8)^2}{(3x+5)^3}$

   (c) $x^3 + y^3 + 6xy = 17$

   (d) $y = \displaystyle\int_2^{\tan(x)} \sqrt{t^3+1}\, dt$

4. Show that the equation $8x - 2\cos(3x+2) = 5$ has exactly one real root. State the theorem(s) that you use in your proof.

5. (a) A ladder is 15 meters long and leaning against a wall. If the top of the ladder slides down the wall at a rate of 0.3 m/s, how fast is the angle between the ladder and the wall changing when the bottom of the ladder is 3 meters from the wall? Express your answer in terms of radians per second.

   (b) If you increase the length of the ladder, but keep everything else the same, will the answer increase, decrease, or remain the same? Explain your answer briefly.

(continued on the back)

6. Let $f(x) = \dfrac{x^2}{x^2 - 9}$.

(a) Find the intervals on which $f(x)$ is increasing and the intervals on which $f(x)$ is decreasing.

(b) Find the local maxima and local minima of $f(x)$, if any.

(c) Find the intervals on which the graph of $f(x)$ is concave up and the intervals on which the graph of $f(x)$ is concave down.

(d) Find the inflection points of the graph of $f(x)$, if any.

(e) Find the horizontal and vertical asymptotes of the graph of $f(x)$.

(f) Using the information found above, sketch the graph of $f(x)$.

7. A rectangular storage container with an open top is to have a volume of 60 m^3. The length of this base is twice the width. Material for the base costs \$5 per square meter. Material for the sides costs \$3 per square meter. Find the cost of materials for the least expensive such container.

8. Use the limit of a Riemann sum to compute the area between the graph of $y = 4 - x^2$ and the x-axis. (Hint: You can use symmetry to focus on the area in the first quadrant.)

Note: $\displaystyle\sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

9. Compute the following integrals:

(a) $\displaystyle\int \frac{4x^3 + 2x - 1}{\sqrt{x}}\,dx$

(b) $\displaystyle\int_{\pi^2}^{4\pi^2} \frac{\sin(\sqrt{x})}{\sqrt{x}}\,dx$

(c) $\displaystyle\int_0^2 \frac{x}{\sqrt{9 + 4x^2}}\,dx$