

QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS

Final Examination

$2\frac{1}{2}$ Hours

Mathematics 143

Spring 2024

Instructions: Answer all questions and show all work

1. Compute:

(a) $\int x^2 \ln x \, dx$

(b) $\int \frac{(x^2 - 1)^{1/2}}{x} \, dx$

(c) $\int \sin^3 x \cos^5 x \, dx$

(d) $\int \frac{4x^2 + x + 5}{(x - 1)(x^2 + 9)} \, dx$

2. Without using a calculator, compute:

(a) $\lim_{x \rightarrow 0} \frac{x^3}{1 - \cos^3 x}$

(b) $\lim_{x \rightarrow 0} [\sec x + \tan x]^{1/x}$

3. Determine if the improper integral $\int_0^{\infty} x e^{-x} \, dx$ converges or diverges, and if it converges, find its value.

4. Determine if each of the following series converges or diverges. You must justify your answer.

(a) $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

(b) $\sum_{n=1}^{\infty} \frac{n^{2/3}}{n^{5/3} + 10}$

(c) $\sum_{n=2}^{\infty} \frac{n(1 + e^{-n})}{n^2 - 1}$ Hint: $1 + e^{-n} > 1$

5. Find the interval of convergence of the series $\sum_{n=1}^{\infty} (-1)^n \frac{(x - 3)^n}{n}$.

6. Using power series, evaluate $\int_0^{0.1} e^{-x^2} \, dx$ with error $< 10^{-8}$

7. Find the Maclaurin series for $f(x) = x^5 \sin(3x)$.

8. Let $f(x) = \ln x$.

(a) Compute $T_3(x)$, the third Taylor polynomial, for $f(x)$ at 2.

(b) Find an expression for $|R_3(x)|$, the error made when using $T_3(x)$ to estimate $f(x)$.

(c) What is the largest possible error we can make if we use $T_3(x)$ to estimate $f(x)$ on the interval $1.5 \leq x \leq 2.5$?