Queens College
Department of Mathematics

Final Examination
$2\frac{1}{2}$ Hours

Mathematics 142 **Spring 2024**

<u>Instructions</u>: Answer all questions. Show work unless the question states otherwise. Partial credit will be awarded for relevant work.

<u>Part I</u>

1. State the Fundamental Theorem of Calculus Part I and Part II. Then use it to compute y'' if

$$y = \int_1^{2\sqrt{x}} \sin t \, dt$$

2. (a) Use the Midpoint (M_n) Riemann Sum with 4 subintervals of equal length to estimate the area under the curve described below for $x \in [0,2]$.

$$y = x^2 - 7x + 12$$

(b) Use the limit of a Riemann Sum Method to compute the exact area under the curve in the region described above.

<u>Part II</u>

1. Compute the following integrals.

(a) $\int \frac{x}{x^2+1} dx$

(b) $\int (2-x)(3+x)\, dx$

(c) $\int \frac{e^{5/x}}{x^2} dx$

(d) $\int \sec^2(4x) \tan^3(4x)\, dx$

(e) $\int \frac{dx}{9+16x^2}$

2. Find the derivative of each of the following functions:

(a) $f(x) = 2^{3x^2} + e^{\cos x} - \log_3(4x+1)$

(b) $g(x) = (\sin x)^x$

(c) $h(x) = \frac{2}{\ln(ax)}$, where a is a non − zero constant

(d) $y = \arcsin(\arctan(3x+5))$

(continued on the back)

3. Suppose $v(t)$ denotes the velocity function (in meters per second) given for a particle moving along a line.

$$v(t) = t^2 - 2t - 8, \qquad 1 \le t \le 6$$

During the given time interval, find

(a) the displacement of the particle.

(b) the distance traveled by the particle.

4. Find an equation of the curve that passes through the point $(1,2)$ if its tangent line at (x,y) has a slope given by

$$\frac{dy}{dx} = \frac{\ln x}{xy} .$$

5. A sample of trititum-3 decayed to 94.5% of its original amount after a year.

(a) What is the half-life of tritium-3?

(b) How long would it take the sample to decay to 10% of its original amount?

6. (a) Find *A*, the area of the region *R* between the two given curves:

$$f(x) = x^2 \quad \text{and} \quad g(x) = 2x - x^2$$

(b) Find the volume, *V*, of the solid obtained by rotating the region *R* above about the line $x = -1$.

7. **Set up** but **DO NOT EVALUATE** the integral that can be used to compute the exact length of the following curve:

$$y = \ln(\sec x) \quad \text{where } x \in \left[0, \frac{\pi}{3}\right]$$