QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS

Final Examination
$2\frac{1}{2}$ Hours

Mathematics 141 **SPRING 2024**

Instructions: For each solution, please *show all of your work* in the blue book provided.

1. Without use of a calculator, algebraically evaluate each of the following limits. If the limit is $+\infty$, $-\infty$ or does not exist, explain why.

   (a) $\lim\limits_{x\to -2} \dfrac{3x^2-3x-18}{4x^2+x-14}$

   (b) $\lim\limits_{x\to 3} \dfrac{\sqrt{5x+1}-4}{x-3}$

   (c) $\lim\limits_{x\to \infty} \dfrac{3x^3-4x+1}{\sqrt{5-x^3+16x^6}}$

   (d) $\lim\limits_{x\to 0} \dfrac{\csc(3x)}{5\cot(2x)}$

2. Using your calculator, create and show a table of values to estimate the value for the following limit. If the limit is $+\infty$, $-\infty$, or does not exist, explain why.

$$\lim_{x\to -1} \frac{\dfrac{4}{x^2}-4}{x+1}$$

3. Consider the following piecewise function.

$$f(x)=\begin{cases} ax+1, & \text{if} \quad x<-1 \\ x^2+2x+6, & \text{if} \quad -1\le x<1 \\ 2x-b, & \text{if} \quad x\ge 1 \end{cases}$$

   Find the values of a and b that make f continuous everywhere.

4. (a) Using only the definition of the derivative, find $f'(x)$ if $f(x)=\sqrt{2x+7}$.

   (b) Use the result found in part (a) to find an equation of the tangent line to the graph of $f(x)$ at the point where $x=-3$.

   (c) Verify that the function f, from part (a), satisfies the hypotheses for the Mean Value Theorem on the interval $[1,9]$, and find the number c that satisfies the conclusion of the Mean Value Theorem.

5. Find $y'=\dfrac{dy}{dx}$ for each of the following: (*Algebraic simplification is not required.*)

   (a) $y=\dfrac{1}{13}x^{52}+15\sqrt[5]{x}-\dfrac{\pi}{x^3}+31^2$

   (b) $y=(20x^4-5x)^3(7x^2+8)^{-4}$

   (c) $y=\dfrac{24\sqrt[4]{x^5}}{\sec(x)-1}$

   (d) $y=(7x^2+\cos^4(3x))^8$

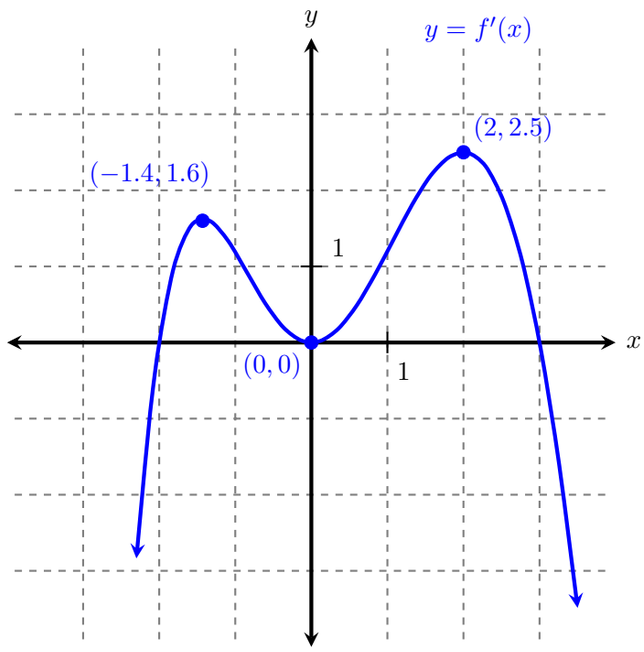
   (e) $\tan(3y)-4xy=5y^2+3x$

(continued on the back)

6. A child, who is in the outdoors holding a balloon, suddenly stops and lets go of the balloon. The balloon rises at a constant speed of 3 ft/s. One second after letting go of the balloon, the child decides to run back home at a speed of 9 ft/s. How fast is the distance between the child and the balloon increasing just 4 seconds after the child decides to run back home? (*Express your final answer rounded to the nearest two decimal places.*)

7. Find the points on the ellipse $4x^2 + y^2 = 16$ that are farthest away from the point $(2, 0)$.

8. The graph of the first derivative, $f'(x)$, of a function $y = f(x)$ is shown below.



Note: The extrema of the graph of $f'(x)$ are indicated by the plotted points with each of their coordinates.

Using calculus and showing all necessary work to justify your answers,

(a) find the critical numbers of f.
(b) find the intervals on which f is increasing and those on which f is decreasing.
(c) find and classify all extrema of f, stating the x-coordinate(s) of the point(s).
(d) find the intervals on which f is concave upward and those on which it is concave downward.
(e) state the $x-$coordinate(s) of the point(s) of inflection of f.
(f) assuming that $f(0) = 1$, use the information in parts (a) through (d) to sketch the graph of $y = f(x)$.