

QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS

Final Examination
2 ½ hours

Mathematics 131

Spring 2024

Instructions: Please answer all questions and show your work in the blue book provided.

1) For the graph of $f(x)$ pictured at the right, please find the following:

a. $\lim_{x \rightarrow -3^-} f(x)$

b. $\lim_{x \rightarrow -3^+} f(x)$

c. $\lim_{x \rightarrow -3} f(x)$

d. $\lim_{x \rightarrow 2^-} f(x)$

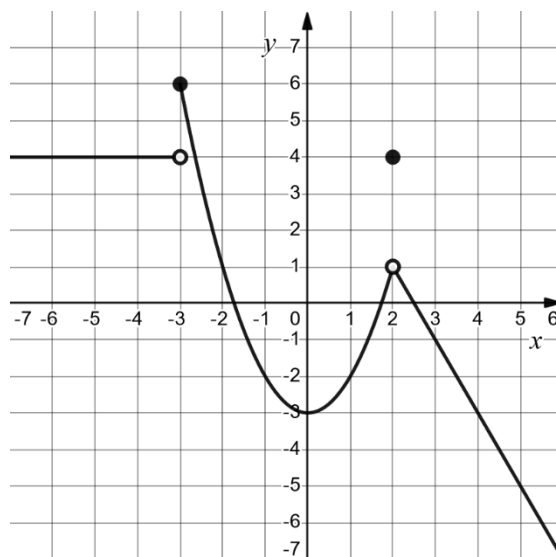
e. $\lim_{x \rightarrow 2^+} f(x)$

f. $\lim_{x \rightarrow 2} f(x)$

g. $f(2)$

h. $\lim_{x \rightarrow -\infty} f(x)$

i. Where is the function f discontinuous? Explain using the definition of continuity.



2) Evaluate the following limits: ($-\infty$, ∞ , and DNE are acceptable possible answers)

a. $\lim_{x \rightarrow 1} \frac{\ln(2-x)}{e^{2x-2}}$

b. $\lim_{x \rightarrow 3^+} \frac{x+2}{3-x}$

c. $\lim_{x \rightarrow +\infty} \frac{5x^3 - 4x}{7x^3 + 2x^2}$

d. $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x^2 - 5x + 6}$

3) Given the function $f(x) = 3x^2 - 6x - 7$.

- a. Use the **definition of the derivative** to find $f'(x)$.
- b. Find an equation of the tangent line at the point $(2, -7)$.

4) Find $\frac{dy}{dx}$ for each of the following. (You do not need to simplify.)

a. $y = \frac{5x^2}{3} - \frac{\sqrt[7]{x}}{x^3} + \frac{7}{e^x} + \ln(10)$

b. $y = \frac{\ln(5x^2 - 1)}{\sqrt{4x - 1}}$

c. $y = (e^{2x} - 8)^{5x^2 - 2}$ (Hint: use logarithmic differentiation)

d. $e^{-y} + 5x^3y + 8 = -x + \sqrt{y^3} + e^4$

(continued on the back)

- 5) Let $f(x) = x^3 - x^2 - 5x + 15$. Using calculus (not the calculator)
- Find the intervals of increase and decrease of f .
 - Find the relative (local) maxima and minima of f and their coordinates.
 - Find the intervals of upward and downward concavity f .
 - Find the inflection point(s) of f , if any
 - Graph $f(x)$ using the information from parts a-d and label all relative extrema and inflection points.
- 6) The demand for x number of units is given by the price function $p(x) = -.0035x^2 + 25$ and the total cost function for x number of units is given by $C(x) = -0.07x^2 + 200$.
- Find the total profit function $P(x)$.
 - Use marginal analysis to estimate the profit associated with the 80th unit.
- 7) A manufacturer estimates that when x units of a certain commodity are produced, the profit obtained is $P(x)$ thousand dollars, where $P(x) = -x^3 + 72x^2 - 1140x - 1675$ for $0 \leq x \leq 45$.
- Can we use the Intermediate Value Theorem to show that we break-even ($P(x) = 0$) at some point in the specified interval? Explain.
 - Find the critical number(s) of the profit function, $P(x)$.
 - Find the number of units that would produce the largest loss (absolute minimum) and the largest profit (absolute maximum) in the specified interval given above. What is the largest loss and profit?
- 8) The demand function for a calculator is given by $p = 48 + \frac{1}{2}x^2$ where x represents the number of calculators (measured in thousands) and p is the unit price measured in dollars. How fast is the quantity of calculators changing when the unit price is \$60.50 and increasing at a rate of \$0.75/month?
- 9) Suppose that \$4200 is invested at the interest rate of 1.85% per year.
- Compute the balance after 10 years if interest is compounded monthly.
 - Compute the balance after 10 years if interest is compounded continuously.
 - How long will it take to reach a balance of \$5500 if interest is compounded continuously?