Mathematics 122   **Spring 2024**
Instructions: Answer all questions. Show all work.

1.   Let $f(x) = \sqrt{x - 3}$.
      (a)   Sketch the graph of $f(x)$ indicating the coordinates of x and y intercepts, if any.
      (b)   Determine the domain and the range of $f(x)$.
      (c)   Use the graph of $f(x)$ to sketch the graph of $f^{-1}(x)$ on the same set of axes, and label x and y intercepts, if any.
      (d)   Find the equation of $f^{-1}(x)$ and determine its domain and range.

2.   Let $f(x) = \sqrt{x - 3}$, and $g(x) = x^2 + 2x + 4$.
      (a)   Compute $f(7) - g(2) + 5$.
      (b)   Find $(f \circ g)(x)$ and write it in simplest form.
      (c)   Find and simplify the difference quotient $\frac{g(a+h) - g(a)}{h}$, $h \neq 0$.

3.   Find the domain of each of the following functions. Express your answer in interval notation.

      (a) $y = x^5 + 2x^3 + 7x - 1$

      (b) $s(x) = \dfrac{x - 5}{3x^2 + 5x - 2}$

      (c) $l(x) = \dfrac{\sqrt{2x - 3}}{x - 5}$

      (d) $r(x) = \dfrac{x - 5}{\sqrt{2x - 3}}$

4.   Sketch the graph of each of the following functions <u>by using transformations</u>. Label the coordinates of any x and y-intercepts. Write equations of the vertical and horizontal asymptotes where appropriate.

      (a) $y = 3 - |x + 2|$

      (b) $y = (x - 1)^3 - 8$

      (c) $y = \dfrac{-12}{x + 3} + 2$

      (d) $y = -e^x - 2$

      (e) $y = \log(x - 3) - 1$

5.   Let $p(x) = -x^2 + 4x - 5$.
      (a)   Write $p(x)$ in standard vertex form by completing the square.
      (b)   What are the coordinates of the vertex?
      (c)   Sketch the graph of $y = p(x)$. Label the vertex and the x and y intercepts, if any.
      (d)   What is the domain of $p(x)$?
      (e)   What is the range of $p(x)$?

6. A man invests \$ 10,000 in an account that pays 8% interest per year, compounded continuously.
   (a) How much money will be in his account after 6 years? (Round answer to the closest cent.)
   (b) How long will it take for his initial investment of \$ 10,000 to triple? (Round answer to one decimal place.)

7. Solve each of the following equations for x. Round your answer to four decimal places when necessary.
   (a) $\log_9(x+3) + \log_9(x-5) = 1$

   (b) $e = 7^{2x-1}$

   (c) $125^x = 25^{(x^2-1)}$

   (d) $5 - \ln(x+2) = 0$

8. Let $r(x) = \dfrac{3x-5}{x+2}$.

   (a) Find $r^{-1}(x)$
   (b) Evaluate $r^{-1}(1)$.

9. Let $y = -\cos(2x) + 1$.
   (a) What is the amplitude of y?
   (b) What is the period of y?
   (c) Sketch the graph of y in the interval $[0, 2\pi]$. Label x and y intercepts.
   (d) What is the range of y?

10. Find the **exact** value **without** the use of a calculator.
    (a) $\csc\left(\cos^{-1}\left(\dfrac{7}{25}\right)\right)$

    (b) $\log_3 36 - \log_3 16 + \log_3 12$

    (c) $\cos\left(\dfrac{-7\pi}{6}\right)$

    (d) $\sin 120° \cos 15° + \sin\ 15°\cos\ 120°$

11. Verify the identity: $\tan x \cdot \sin x + \cos x = \sec x$

12. If $\sin(A) = -\dfrac{12}{13}$ where A is in Quadrant IV, and $\tan(B) = \dfrac{4}{3}$ where B is in Quadrant III, evaluate the following:

    (a) $\sec(B)$

    (b) $\cos(A - B)$

    (c) $\sin(2B)$

    (d) $\cot(A)$

13. Solve for x in the interval $[0, 2\pi)$.
    $$2 \sin^2 x + \cos x - 1 = 0$$