MATH 120 — Final Exam — 21 May 2024

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Class Section: _____

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Important Information:

WAIT until you are told to start.

For the counting questions, leave your answers in terms of $n!$, $P(n,k)$, $\binom{n}{k}$, and $\left(\!\binom{n}{k}\!\right)$.

You are expected to SHOW YOUR WORK in all answers.
Wrong answers provided without work will receive no credit.

You may use the back of pages if you need more space or for scrap work.

If you continue work in a different location and want it to be graded, indicate CLEARLY where it is.

1. Consider the mathematical notation '$\subset$':

   (a) How would you translate the mathematical statement $A \subset B$ into the English language?

   (b) Describe in your own words what the mathematical statement $A \subset B$ means.
   In your description, make sure to use the word 'element'.

2. For each of the following expressions, determine whether it is True or False.

   No explanation is necessary. However, if you add a sentence of explanation you may receive partial credit in case you choose the wrong answer and convey an understanding of the situation.

   (a) $\varnothing \in \{-1, 0, 1\}$.

   (b) $\mathbb{R}$ and $\mathbb{Z}$ are disjoint sets.

   (c) $\mathbb{N}$ is a finite set.

3. Write the set $\{\cdots, -14, -10, -6, -2, 2, 6, 10\}$ in set-builder notation.

4. Let $V = \{1, \ldots, 20\}$ and $W = \{(v, v, v) : v \in V\}$. Is $W = V \times V \times V$? Explain your reasoning.

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5. Let A be the set of people who like **both** pasta **and** chicken.
   Describe in words the set of people **NOT in** A, without using the word "and".
   [*Hint: You may find it helpful to consider De Morgan's Law.*]

6. How many six-letter passwords use either numbers (0-9) or capital letters (A-Z) but not both?
   [*No other characters are allowed in the passwords: no lower case letters, no symbols, etc.*]

7. You go to your local library. It has 1000 books, all with different titles.

   (a) In how many ways can you choose ten books to take home?

   (b) In how many ways can you put those ten books on a shelf **in alphabetical order**?

8. You decide to bake sugar cookies with frosting for the fourth of July. You have three different colors of frosting: red, white, and blue.

   (a) How many combinations of frosting colors can you put on one cookie? (Assume there **must** be frosting on the cookie and that you cannot mix colors to make new colors.)

   (b) Now assume you put only one color of frosting on each cookie. In how many ways can you color 50 cookies if there must be at least 10 of each color?

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9. Consider the rule r that takes as input a shirt and gives as output the person wearing the shirt. Explain why r is not a well defined function.

10. Consider the function p that takes as input an item in your local grocery store and gives as output the price in dollars.

    (a) Determine the domain of p.

    (b) Determine a possible codomain for p.

    (c) What is $p^{-1}(5)$? Describe this quantity completely.

11. Consider the function $f : \mathbb{N} \to \mathbb{N}$ that computes $f(x) = \lfloor x/2 \rfloor$.

    (a) Is f injective? Explain your answer.

    (b) Is f surjective? Explain your answer.

12. Apply the rules of logarithms and exponents to simplify the following expressions as directed.

    (a) Write $\log(1000)/\ln(e^2)$ without logarithms.

    (b) Write $\dfrac{4^x 8^y}{\sqrt{2} + 3\sqrt{2}}$ as an expression of the form $(\text{something})^{(\text{something})}$.

    (c) Write $\log_3(x^4)$ as an expression only involving terms of the form $\log_5(\text{something})$.

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13. Compute terms a_0 through a_5 of the sequence $a_n = (n \,\%\, 3) + (n \,//\, 2)$

14. Write the sum $16 + 12 + 9 + \frac{27}{4} + \cdots$ in sigma notation. Do not compute the sum.

15. Compute $\sum_{i=0}^{100}(i + 5^i)$. Your final answer should not include summation notation.

16. Compute $\prod_{i=0}^{4}(i^2 + 1)$. Write your final answer as an integer, not as a product.

17. Rewrite the following exponential expression as an expression involving product notation.

$$100^{\left(\sum_{k=a}^{b} k^2\right)}$$

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18. Suppose it is currently Tuesday at 5:00 pm. What day and time will it be 777 hours from now? Explain your reasoning.

19. Use the Euclidean Algorithm to find $\gcd(1155, 504)$.

20. Find the prime factorization of 910000.

21. Suppose that p, q, and r are distinct primes and that $m = p^{100}q^{200}$ and $n = p^{150}r^{50}$.

    (a) How many divisors does m have?

    (b) Determine $\gcd(m, n)$.

    (c) Determine $\text{lcm}(m, n)$.

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