QUEENS COLLEGE
MATHEMATICS DEPARTMENT

FINAL EXAM
$2\frac{1}{2}$ HOURS

Math 152 **Fall 2016**

INSTRUCTIONS: ANSWER ALL QUESTIONS SHOW ALL WORK

1. a) Let R_1 be the region bounded by the curve $y = \frac{1}{x}$, $y = 0$ and the lines $x = 1$ and $x = 4$.
   Set up but do not evaluate the integral needed to find the volume of the region when R_1 is rotated about the line
   i) $y = -1$
   ii) $x = 7$

   b) Let R_2 be the region bounded by $y = \frac{1}{x}$, the x-axis and $x \geq 1$.
   i) Show that the area of the region R_2 described above is infinite. (Hint: use an improper integral.)
   ii) Show that the volume of the solid of revolution obtained by rotating R_2 about the x-axis is finite and compute it. (Hint: use an improper integral.)

2. In each of the following find the derivative. (Algebraic simplification is unnecessary.)

   a) $y = e^{\tan^{-1} x} \ln(7^x)$

   b) $y = \sin^{-1}(e^{4x})$

3. Find the limit if it exists. If it does not exist, explain why.

   a) $\lim_{x \to 0} \dfrac{x - \tan^{-1} x}{x^3}$

   b) $\lim_{x \to 1} x^{\frac{1}{1-x}}$

4. Evaluate the following integrals. If an integral does not converge, explain why.

   a) $\int x^2 e^{2x}\, dx$

   b) $\int \dfrac{\sqrt{x^2+9}}{x^4}\, dx$

   c) $\int \dfrac{x\, dx}{(x+1)(x^2+1)}$

   d) $\int_0^2 \dfrac{x}{x^2-1}\, dx$

(continued on the back)

5. Bacteria in a culture increase at a rate proportional to the number of bacteria present. An initial population of 300 triples in 10 hours. If this pattern continues, find the number of bacteria present after one day.

6. Solve the initial value problem:

$$t\frac{dx}{dt} = x\ln x \quad , \quad x(1) = e$$

7. Determine whether the series is conditionally convergent, absolutely convergent or divergent.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin n}{n^2+1}$

b) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n \ln n}{n}$

c) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n!\, n}$

8. Find the radius and interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{2^n(x-3)^n}{n^2}$$

9. a) Using a series representation for $f(x) = \arctan x$, find a series representation for $\frac{\arctan(x^2)}{x}$.

b) Use the series found in part a) to approximate the value of

$$\int_0^{\frac{1}{2}} \frac{\arctan(x^2)}{x}\, dx$$

with $R_n \leq 0.001$.

10. Let $f(x) = \ln(2x)$

a) Find T_4 , the fourth Taylor polynomial centered at $a = 1$.

b) Use part a) to approximate the value of $\ln(2.2)$ correct to 3 decimal places.
(Hint: $\ln(2.2) = \ln\big(2(1.1)\big)$.)

c) Find an estimate for the error in part b) using Taylor's Theorem.