QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS
FINAL EXAMINATION
$2\frac{1}{2}$ HOURS

Mathematics 143 **Spring 2022**

Instructions: Answer all questions. Show all work.

1. Evaluate the limit: $\lim_{x \to 0}\left(x + e^{\frac{x}{2}}\right)^{\frac{3}{x}}$

2. Integrate the following:

a) $\int \frac{\ln x}{x^4}\,dx$

b) $\int \frac{dx}{x^3\sqrt{x^2 - 9}}$

c) $\int \frac{9x^4 + 9}{x^3 + 9x}\,dx$

d) $\int_0^{\infty} \frac{e^x}{e^{2x} + 1}\,dx$

e) $\int_{-1}^{8} \frac{1}{x^{1/3}}\,dx$

3. Determine if the sequence $\{a_n\}$ converges or diverges. Find the limit if it converges.

a) $a_n = \left(1 - \frac{3}{n}\right)^n$

b) $a_n = \frac{n^3 + 4n - 5}{n^2 - 4n - 5n^3 + 2}$

4. Determine if the series converges or diverges. State what test you used.

a) $\sum_{n=1}^{\infty} \frac{\pi^{n+1}}{5^n}$

b) $\sum_{n=1}^{\infty} \frac{3n - 2}{n \cdot 5^n}$

5. Determine if each of the following series converges absolutely, converges conditionally, or diverges. State which test you use in each case.

a) $\sum_{n=1}^{\infty} \frac{(-1)^n\, n}{n^2 + 3}$

b) $\sum_{n=1}^{\infty} (-1)^{2n-1} \frac{\pi^{2n+1}}{(2n + 1)!}$

(continued on the back)

6. Find the radius and interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n (x+3)^n}{n \cdot 4^n}$.

7. Given $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$.

a) Find the Maclaurin series representation for the function $f(x) = x^3 \tan^{-1}(x^2)$.

b) Use the result of a) to approximate the definite integral $\int_0^{1/2} x^3 \tan^{-1}(x^2)\, dx$ with five decimal place accuracy.

8. Let $f(x) = x^{3/2}$.

a) Write the fourth degree Taylor polynomial, $T_4(x)$, for $f(x)$ centered at $a = 4$.

b) Use Taylor's formula to estimate the accuracy when $T_4(x)$ is used to approximate $f(4.1)$.